Effects of short-range correlations on proton densities using the Wood-Saxon potential

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Jastrow approach has long been used to describe short-range correlations within the nucleus. But unless the number of nucleons in the nucleus is very small, it is not possible to calculate the proton density and form factor exactly by employing the Jastrow wave function. To deal with this case, Iwamoto and Yamada have developed the Cluster Expansion method [1] and M. GAUDIN et al. have applied that method to nuclei with simple correlation factor $e^{-\beta^2 r^2}$ [2]. Here we first discuss a new method [3] to calculate the effect on the proton density caused by short-range correlation that can be applied to heavier nuclei. In this report we will provide the result produced by this method for the proton density, especially in the high Z, N≠Z method.

Under the independent particle approximation, the shell model many body wave function is given by the Slater determinant of the occupied single particle wave functions $\psi_i(\boldsymbol{r_j})$:

$$\psi_{SM} = \frac{1}{\sqrt{A!}} \det\left(\psi_1(\boldsymbol{r_1}) \ldots \psi_A(\boldsymbol{r_A})\right)$$

To account for the short range correlation, we use Jastrow wave function:

$$\psi_{corr} = \frac{N}{\sqrt{A!}} \prod_{1\le i<j}^{A} f_{ij}(|\boldsymbol{r_i} - \boldsymbol{r_j}|) \det\left(\psi_1(\boldsymbol{r_1}) \ldots \psi_A(\boldsymbol{r_A})\right)$$

In which N is normalize factor. In long range limit f_{ij} goes to 1.

Therefore, we can assume that for g_{ij}=f_{ij}-1, which is not negligible only when $\boldsymbol{r_i} - \boldsymbol{r_j}$ is small, we can use,

$$\prod_{1\le i<j}^{A} f_{ij}(|\boldsymbol{r_i} - \boldsymbol{r_j}|) = \prod_{1\le i<j}^{A} (1 + g_{ij}(|\boldsymbol{r_i} - \boldsymbol{r_j}|))$$

We also note that the chance that more than 3 nucleons are all close to each other is small, and therefore we can neglect all the diagrams [3] that involves more than 3 nucleons. Thus, we can get for the correlated density ρ(**r**)

$$\rho(\mathbf{r}) = \rho0(\mathbf{r}) + \int g(\mathbf{r} - \mathbf{r1})\,(\rho0\mathrm{sum}(\mathbf{r1})\rho0(\mathbf{r}) - \rho0(\mathbf{r},\mathbf{r1})\rho0(\mathbf{r1},\mathbf{r}))d\mathbf{r1} - \int g(\mathbf{r1} - \mathbf{r2})\left(\rho0(\mathbf{r},\mathbf{r1})\rho0(\mathbf{r1},\mathbf{r})\rho0\mathrm{sum}(\mathbf{r2}) - \rho0(\mathbf{r},\mathbf{r1})\rho0(\mathbf{r1},\mathbf{r2})\rho0(\mathbf{r2},\mathbf{r}')\right)\, d\mathbf{r1}d\mathbf{r2}, \quad (1)$$

where ρ0(**r**) and $\rho0(\mathbf{r},\mathbf{r1})$ are the shell model one-body density matrix for proton or neutron with a certain spin state, and $\rho0\mathrm{sum}(\mathbf{r1})$ is the sum of the density of both kinds of nucleons (proton and neutron of all spin states) within the shell model.

Under the method that we developed, if we set $g(\boldsymbol{r_i}-\boldsymbol{r_j})$=$-e^{-\beta^2|r_i-r_j|^2}$ we can calculate the nucleon density distribution for all the spherical symmetric nucleus, especially heavier one. How well it works? Here is an example:

We use the Wood-Saxon potential: V=($\frac{50}{1+e^{\frac{r-1.25A^{\frac{1}{3}}}{(0.6-1.2A)fm}}}(1+0.72\frac{N-Z}{A})$+$V_{columb}$) MeV, in which V_{columb}=1.44Z/r MeV for r>r_o=$1.25A^{\frac{1}{3}}$ and 1.44Z($\frac{3r_0^2-r^2}{2r_0^2}$) MeV for r<$r_o$, for neutron the potential is V=($\frac{50}{1+e^{\frac{r-1.25A^{\frac{1}{3}}}{(0.6-1.2A)fm}}}(1+0.72\frac{Z-N}{A})$) MeV. For the correlation factor we use $g(\boldsymbol{r_i}-\boldsymbol{r_j})$=$-e^{-1.4^2|r_i-r_j|^2}$. The calculated proton root-mean-square (RMS) radii are given in Table I and the shell model and the correlated proton density distributions are shown in Figs 1 and 2.

Table I. Calculated proton root-mean-square (RMS) radii.

nucleus	4He	16O	28Si	32S	40Ca	60Ni	90Zr	140Ce	208Pb
shell model radius	1.616	2.530	3.059	3.190	3.330	3.756	4.150	4.787	5.448
radius with correlation(fm)	1.737	2.677	3.182	3.330	3.484	3.857	4.283	4.905	5.542
radius change	7.47%	5.81%	4.02%	4.37%	4.62%	2.66%	3.19%	2.46%	1.73%

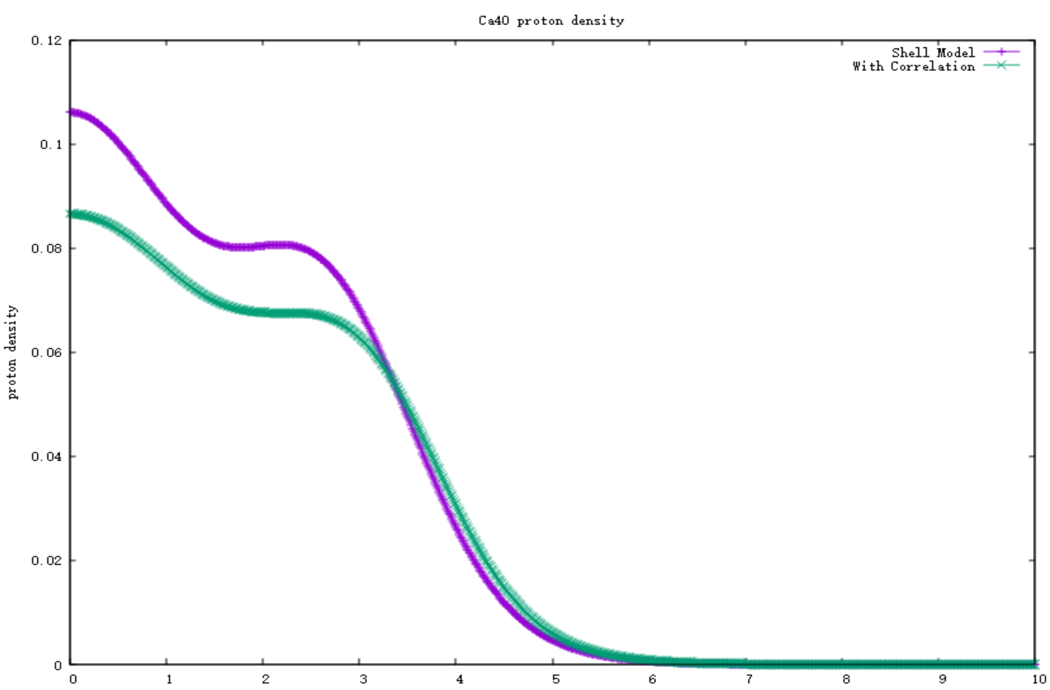

FIG. 1. The shell model (red line) and the correlated (blue line) proton density distributions of ^{40}Ca.

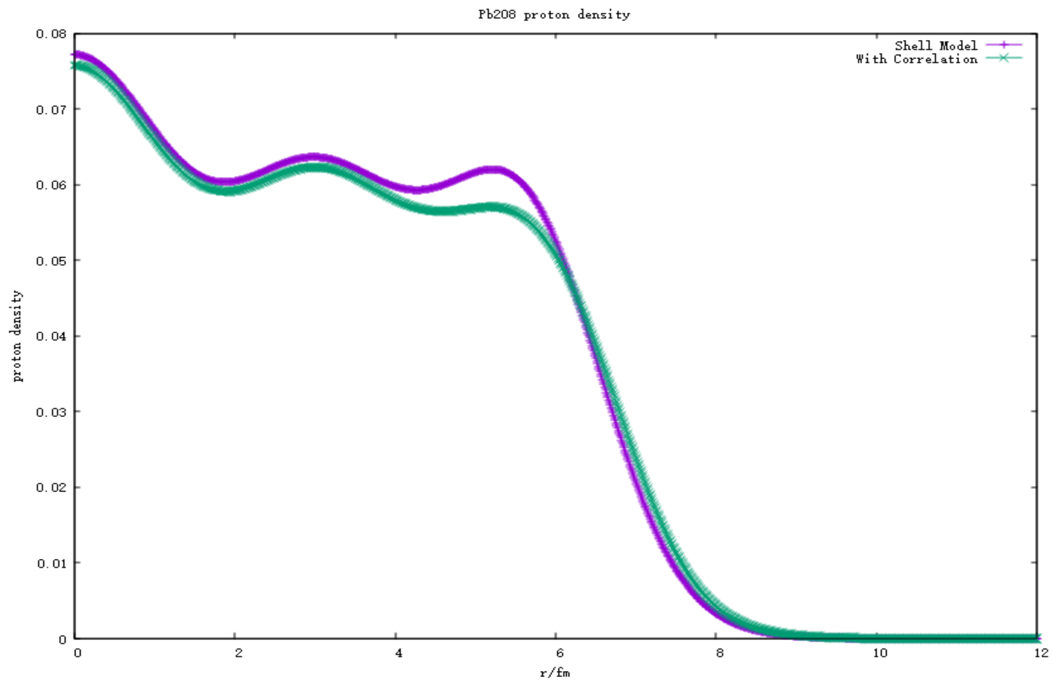

FIG. 2. The shell model (red line) and the correlated (blue line) proton density distributions of ^{208}Pb.

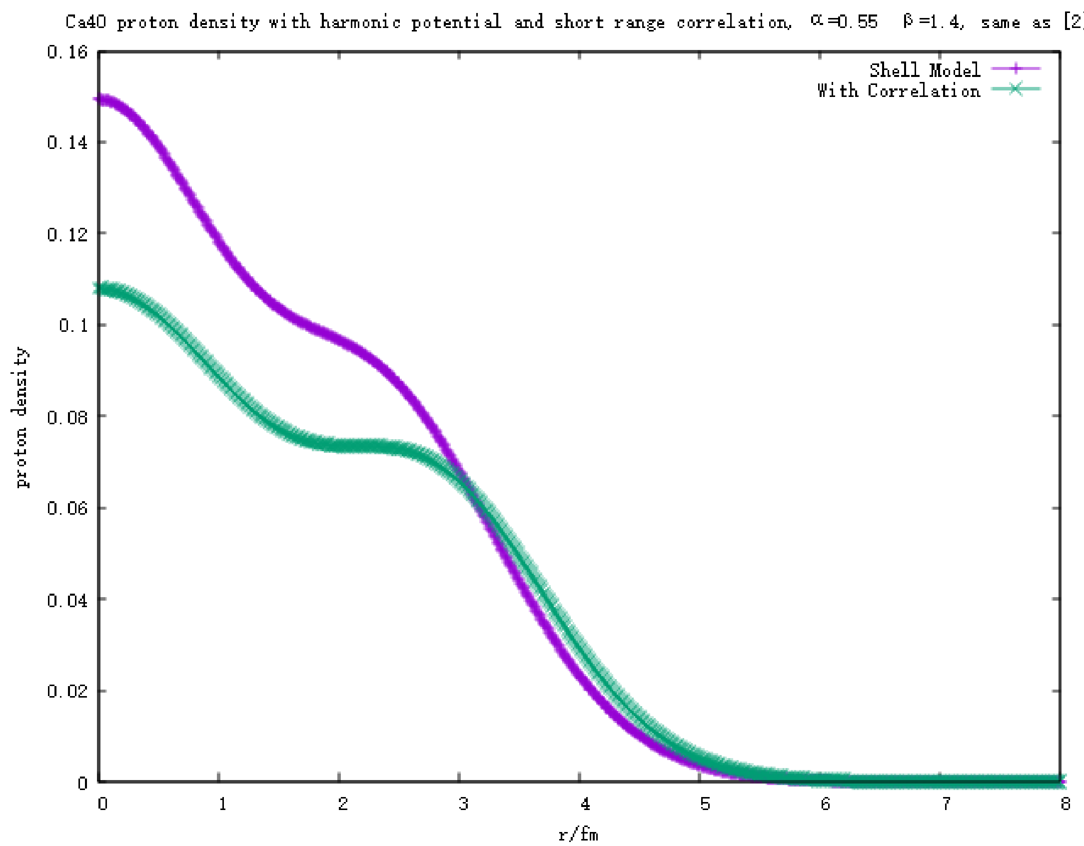

FIG. 3. Proton density for ^{40}Ca with harmonic potential [2] and short range correlation.

In order to confirm the correctness of my calculaion, I used my method for the case provided in [2]: Ca40 with harmonic shell model α=0.55fm-1 and short-range correlation factor β=1.4fm-1, and the result withCorrelation is shown in Fig. 3.

[1] F. Ywamoto and M. Yamada, Progr. Theor. Phys. **17**, 543 (1957).
[2] M. Gaudin, J. Gillespie, G. Ripka, Nucl. Phys. **A176**, 217 (1971).
[3] Tianyang Ma and Shalom Shlomo (to be published).